It was shown above that the energy flux of the wave is conserved. This is related to the neglect of the wave attenuation. A weak attenuation can be introduced in Eq. (3.2) in analogy to hydrodynamics. This fact and the consideration that the dispersion equation for waves in a low-viscosity fluid is described by (3.3) makes it possible to determine the total change in amplitude related to ray focusing and wave attenuation.

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EXCITATION OF THERMAL-CAPILLARY CONVECTION AT A DEFORMABLE INTERFACE IN SYSTEMS WITH A SURFACE-ACTIVE AGENT

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The thermal-capillary instability of the layer of a liquid with a free surface has been studied, on which a surface-active agent has been applied [1, 2]. The problem of the initiation of thermal-capillary convection in the presence of a surface-active agent has been solved [3, 4] in the two-layer formulation with a consideration of the hydrodynamic and thermal processes on both sides of the separation surface. In all cases, the problem was examined under the assumption of a plane undeformed interface. It is known that interface deformation can have a significant effect on the excitation of thermal-capillary convection [5-7].

Here, the instability of the equilibrium of systems containing surface-active agents is investigated with a consideration of the deformation of the interface. The effect of the surface-active agent is studied on the monotonic instability mode, and also on oscillatory modes of various types. Features are explained for exciting a special type of oscillatory instability, which is closely related to the presence of a surface-active agent when the interface is deformed.

1. Let the space between two horizontal solid plates at $y = a_1$ and $y = -a_2$, over which a temperature difference θ is maintained, be filled with two layers of immiscible viscous fluids. The equation of the interface is $y = 0$ in the state of mechanical equilibrium. The densities of the media are ρ_m, the coefficients of dynamic and kinematic viscosity are η_m and ν_m, the thermal conductivities are $\varkappa_m$, and the heat transfer coefficients are χ_m ($m = 1$ for the upper layer and $m = 2$ for the lower one). We assume that a surface-active agent is concentrated with a surface (mass) concentration Γ at the interface. The concentration of

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the surface-active agent is small, so that its molecules form a "surface gas." The dependence of the surface tension on the temperature T and the surface-active agent concentration is assumed linear

$$\sigma = \sigma_0 - \alpha T - \alpha_s \Gamma \ , \tag{1.1}$$

where σ_0, α, and α_s are constants.

In the state of mechanical equilibrium, the surface-active agent concentration is constant: $\Gamma = \Gamma_0$. The excitation of thermal-capillary convection inevitably leads to a non-uniform concentration of the surface-active agent along the interface. The transport equation for exciting a concentration Γ_d [8] takes the form

$$\frac{\partial \Gamma_d}{\partial t} + \Gamma_0 \frac{\partial v_x}{\partial x} = D_0 \frac{\partial^2 \Gamma_d}{\partial x^2} \tag{1.2}$$

if adsorption and desorption are neglected after linearization and if the motion is assumed planar. Here v_x is the horizontal component of the velocity on the interface, and D_0 is the surface diffusion coefficient of the surface-active agent. Because a unit surface area of the interface has a mass Γ_0, the stress balance at the interface, considering (1.1), can be written in the form

$$\Gamma_0 \frac{\partial v_i}{\partial t} = -(p_1 - \rho_1 gh) n_i + (p_2 - \rho_2 gh) n_i - \frac{\sigma}{R} n_i + (\sigma_{1,ik} - \sigma_{2,ik}) n_k - \\ - \alpha D_i T - \alpha_s D_i \Gamma_d, \tag{1.3}$$

where v_i and T are the velocity and temperature at the interface; h is the displacement of the interface; n_i is the normal vector; R is the radius of curvature of the surface; and $\sigma_{m,ik} = \eta_m(\partial v_{m,i}/\partial x_k + \partial v_{m,k}/\partial x_i)$ is the viscous stress tensor of the m-th liquid, the pressure p_m in the m-th liquid is reckoned from the hydrostatic pressure; and $D_i = \partial/\partial x_i - n_i n_k \partial/\partial x_k$ is the surface gradient.

We choose the notation $\rho = \rho_1/\rho_2$, $\eta = \eta_1/\eta_2$, $\nu = \nu_1/\nu_2$, $\varkappa = \varkappa_1/\varkappa_2$, $\chi = \chi_1/\chi_2$ and $a = a_2/a_1$. We introduce a_1, a_1^2/ν_1, ν_1/a_1, $\rho_1\nu_1^2/a_1$, θ and Γ_0 as the units of length, time, velocity, pressure, and concentration. The dimensionless gradients of temperature A_m and pressure B_m are constant at equilibrium and are $A_1 = -s/(1 + \varkappa a)$, and $A_2 = -s\varkappa/(1 + \varkappa a)$, respectively, where s = 1 for heating from above and s = −1 for heating from below; $B_1 = -\mathrm{Ga}$, and $B_2 = -\mathrm{Ga}/\rho$, where $\mathrm{Ga} = g \cdot a_1^3/\nu_1^2$ is the Galileo number. The linearized convection equations for the normalized perturbations of the x and y components of velocity v_{mx} and v_{my}, the pressure p_m, and temperature T_m (m = 1, 2) take the form

$$-(\lambda + i\omega) v_{my} = -e_m p_m' + c_m D v_{my}, \quad -(\lambda + i\omega) v_{mx} = -ike_m p_m + c_m D v_{mx},$$
$$-(\lambda + i\omega) T_m + A_m v_{my} = \frac{d_m}{\mathrm{Pr}} D T_m, \quad ikv_{mx} + v_{my}' = 0.$$

Here k is the wave number; $\lambda + i\omega$ is the complex decrement; the prime denotes differentiation with respect to the y coordinate; $D = d^2/dy^2 - k^2$; $c_1 = d_1 = e_1 = 1$; $c_2 = \nu^{-1}$; $d_2 = \chi^{-1}$; $e_2 = \rho$; and $\mathrm{Pr} = \nu_1/\chi_1$ is the Prandtl number.

The conditions on the solid boundaries are y = 1: $v_1 = 0$, $T_1 = 0$; $y = -a$: $v_2 = 0$ and $T_2 = 0$. On the interface, besides the transport equation for the surface-active agent (1.2) and the conditions for the normal and tangential stresses (L.3), we have the continuity conditions for the velocity vector, the temperature, and the normal heat flow components, and also the kinematic relationships, which relate the displacement h of the interface with the velocity of the fluids on the separation surface. The conditions for deformation of the interface due to transport at the plane y = 0 take the form

$$y=0:\ p_1-p_2+\left[\mathrm{Ga}\left(\rho^{-1}-1\right)+Wk^2\right]h-(\lambda+i\omega)K\eta^{-1}v_{1y}= \\ =2\left(v_{1y}'-\eta^{-1}v_{2y}'\right),$$

$$\eta\left(v_{1x}'+ikv_{1y}\right)-\left(v_{2x}'+ikv_{2y}\right)-ik\,\mathrm{Mr}\left(T_1-\frac{s}{1+\varkappa a}h\right)-ikB\Gamma+ \\ +(\lambda+i\omega)Kv_{1x}=0;\ v_{1x}=v_{2x},\ -(\lambda+i\omega)h=v_{1y}=v_{2y},$$

$$T_1-T_2=\frac{s(1-\varkappa)}{1+\varkappa a}h,\ \ \varkappa T_1'-T_2'=0,\ \ (\lambda-D_sk^2)\Gamma=ikv_{1x},$$

where $W=\sigma a_1/\eta_1\nu_1$; $\mathrm{Mr}=\alpha\theta a_1/\eta_2\nu_1$ is the analog to the Maragoni number; $B=\alpha_s\Gamma_0 a_1/\eta_2\nu_1$; $K=\Gamma_0\eta_1/\rho_1 a_1\eta_2$; and $D_s=D_0/\nu_1$.

It is easy to understand that the parameter K is proportional to the ratio of the mass of surface-active agent particles concentrated on the interface to the mass of the first fluid. Hereafter we will set this parameter to zero.

By introducing perturbation for the flow functions $v_{mx}=\psi_m'$ and $v_{my}=-ik\psi_m$ $(m=1,2)$ and eliminating p_m $(m=1,2)$ and Γ, we write the resultant boundary problem as

$$(\lambda+i\omega)D\psi_m=-c_mD^2\psi_m,\ -(\lambda+i\omega)T_m-ikA_m\psi_m=\frac{d_m}{\mathrm{Pr}}DT_m,$$

$$y=1:\ \psi_1=\psi_1'=T_1=0;\ y=-a:\ \psi_2=\psi_2'=T_2=0,$$

$$y=0:\ \psi_1'''-\eta^{-1}\psi_2'''+[(\lambda+i\omega)(1-\rho^{-1})-3k^2(1-\eta^{-1})]\psi_1'+ik[\mathrm{Ga}(\rho^{-1}-1)+ \\ +Wk^2]h=0,$$

$$\eta\left(\psi_1''+k^2\psi_1\right)-\left(\psi_2''+k^2\psi_2\right)-ik\,\mathrm{Mr}\left(T_1-\frac{s}{1+\varkappa a}h\right)+\frac{k^2B}{\lambda-D_sk^2}\psi_1'=0,$$

$$\psi_1'=\psi_2',\ \psi_1=\psi_2=-i\frac{(\lambda+i\omega)}{k}h,\ T_1-T_2=\frac{s(1-\varkappa)}{1+\varkappa a}h,\ \varkappa T_1'-T_2'=0.$$

2. The interface for a monotonic instability can be determined in an analogous manner:

$$\mathrm{Mr}(k)=\frac{8sk^2(1+\varkappa a)(\varkappa D_1+D_2)(\eta B_1+B_2+B/2kD_s)}{\varkappa\left[\mathrm{Pr}(\chi E_2-E_1)-8k^5(D_1+D_2)\left(F_1-\eta^{-1}F_2\right)\right][\mathrm{Ga}(\rho^{-1}-1)+Wk^2]^{-1}}. \tag{2.1}$$

Here

$$D_1=\frac{C_1}{S_1};\ D_2=\frac{C_2}{S_2};\ B_1=\frac{S_1C_1-k}{S_1^2-k^2};\ B_2=\frac{S_2C_2-ka}{S_2^2-k^2a^2};$$

$$E_1=\frac{S_1^3-k^3C_1}{S_1\left(S_1^2-k^2\right)};\ E_2=\frac{S_2^3-k^3a^3C_2}{S_2\left(S_2^2-k^2a^2\right)};\ F_1=\frac{1}{S_1^2-k^2};\ F_2=\frac{a^2}{S_2^2-k^2a^2};$$

$$S_1=\mathrm{sh}\,k;\ S_2=\mathrm{sh}\,ka;\ C_1=\mathrm{ch}\,k;\ C_2=\mathrm{ch}\,ka.$$

From Eq. (2.1) it can be seen that the presence of the surface-active agent always leads to a displacement of the neutral curve to the side of the larger Mr, which is determined by the combination B/D_s. Because the quantity D_s is usually small for real surface-active agents, this displacement is significant, even for moderate values of B. We note, that the position of the break of the neutral curve, as determined by a zero denominator, does not change in the presence of a surface-active agent.

As $k\to\infty$, Eq. (2.1) transforms to the formula found in [3] without considering the interface deformation. For long-wavelength perturbations, the distortion of the interface has a defined value. For k = 0, the threshold for exciting convection is described by the expression

$$\mathrm{Mr}(0)=\frac{2s\eta\,\mathrm{Ga}\,\delta\,(1+\eta a+Ba/4D_s)\,(1+\varkappa a)^2a}{(1+a)\,(1-\eta a^2)}. \tag{2.2}$$

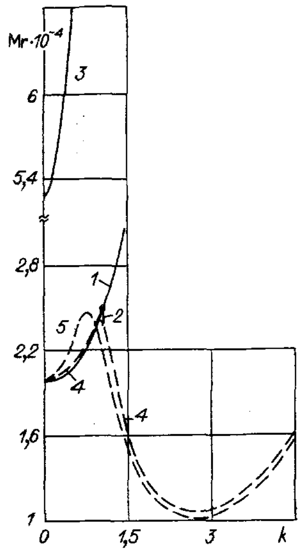

Fig. 1

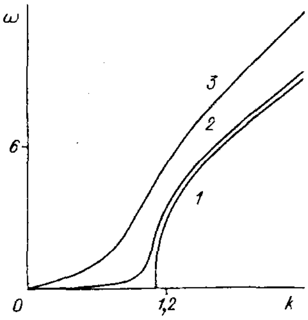

Fig. 2

3. We now study oscillatory type instabilities. The oscillatory neutral curves were calculated numerically by the Runge—Kutta method. We will examine a model system: $\nu = 0.5$, $\varkappa = \chi = a = \mathrm{Pr} = W = 1$, and $\rho = 0.999$. The choice of system parameters is dictated by the following considerations. In the absence of deformation and a surface-active agent, it is unstable relative to oscillatory perturbations when heated from below ("longitudinal" oscillations), and this instability mode is the only one. It is natural to expect that the effect of deforming the interface will be strongest for values of ρ close to unity.

Initially we examine the case of heating from below. If there is no surface-active agent ($B = 0$) but if the interface is deformed, the oscillatory instability is conserved for wave numbers larger than the critical value; in the long wavelength region, the monotonic perturbation is the most critical. The neutral curves 1 and 2, computed for $\mathrm{Ga} = 10^7$, are shown in Fig. 1. (The monotonic curves are solid; the oscillatory curves are dashed.) The dispersion curve 1 for the oscillatory mode of instability is shown in Fig. 2.

The appearance of a surface-active agent ($B \neq 0$), as in the case of no interface deformation [3], splits the monotonic neutral curve in two: a monotonic curve with a threshold value of Mr, in accordance with Eq. (2.2), which grows rapidly with increasing B, and an oscillatory curve, for which the dependence on B is much weaker. Figure 1 shows the monotonic and oscillatory neutral curves 3 and 4 for $B = 0.1$ and the oscillatory neutral curve 5 for $B = 5$. The dependence of the frequency ω on the wave number k is shown in Fig. 2 ($B = 0.1$ and 5 for curves 2 and 3).

When heating is from above, deformation of the interface leads to a new type of oscillatory instability ("transverse" oscillations). The neutral curve has the shape of a sack (see Fig. 3a, $\mathrm{Ga} = 10^4$, $B = 0$, 5, and 15 for curves 1-3); that is, two values of sMr correspond to any value of the wave number in the region $k < k_1$ (B). The corresponding dispersion curves are shown in Fig. 3b (the curve numbers are the same as in Fig. 3a). As B increases, the width of the instability region for a wave number k_1 decreases monotonically, and the threshold value Mr increases.

4. Now we examine a system of real media: air—water for $\mathrm{Pr} = 0.758$, $\eta = 0.0182$, $\nu = 15.077$, $\varkappa = 0.0369$, $\chi = 138.42$, and $\rho = 0.00121$. Let $a = 1$. By assuming the possibility of changing the value of g (under conditions of reduced gravity), we will examine Ga and W as independent parameters. We set $W = 10^6$ (which corresponds to each layer being 3 mm thick) and vary the parameter Ga.

The neutral curves for heating from below are shown in Fig. 4 ($\mathrm{Ga} = 0$) and Fig. 5 ($\mathrm{Ga} = 10$). In both figures the curves 1-4 correspond to $B = 0$, 1, 5, and 10. In the absence of a surface-active agent ($B = 0$), the instability has a monotonic character; if the values of Ga are not too large, the neutral curve has two minima in the short and long wavelength regions. For $B \neq 0$, the neutral curves split into a monotonic curve and an oscillatory curve; the monotonic mode of the instability rapidly stabilizes. (In the figures the monotonic neutral curve is not shown, because it lies at larger values of Mr.)

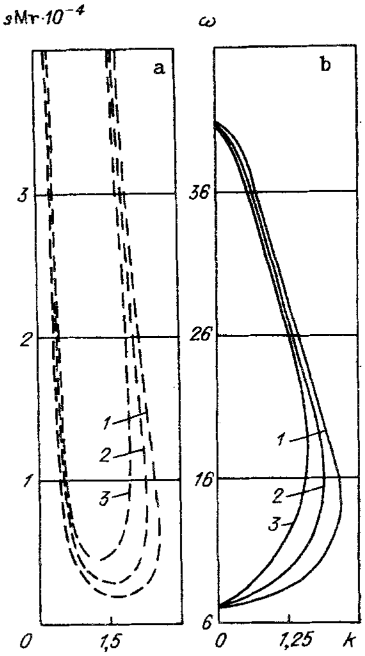

Fig. 3

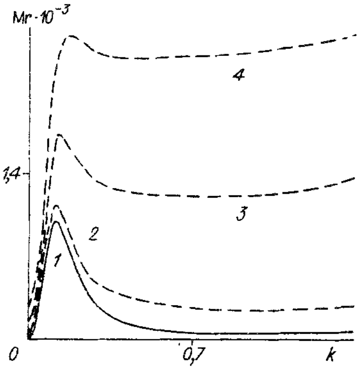

Fig. 4

As B increases, the critical Mr increases for both the long and short wavelength minima. However, the short-wavelength ("longitudinal") oscillations stabilize much faster than the long-wavelength ("transverse") ones. Therefore a situation is possible where for values $B < B_*$ the most critical instability is the short wavelength one, but for $B > B_*$ it is the long wavelength mode (Fig. 5). The oscillation frequency increases monotonically as B increases (Fig. 6, curves 1-3 for B = 1, 5, and 10).

When the air–water system is heated from above, a long-wavelength oscillatory instability occurs, which is closely related to the interface deformation, as in the model system discussed in paragraph 3. The neutral curves are shown in Fig. 7, and the dependence of frequency on the wave number is shown in Fig. 8. In both figures, curves 1-3 correspond to B = 0, 5, and 15. Now we turn attention to the fact that for this system (as opposed to the model situation) the neutral curve for the oscillatory instability does not have the form of a sack. As B increases, the threshold value of Mr and the oscillation frequency increase.

5. As can be seen from the interface condition for normal stresses (and also the results from paragraph 4), when $W \gg 1$, the interface deformation is substantial only in the long wavelength region ($k \ll 1$). In this case, the conditions for exciting an oscillation instability for heating from below can be investigated analytically.

First we examine the wave numbers $k \lesssim W^{-1/2}$. If we expand the solution and the critical values of Mr and ω^2 in a series in k, we find to zero order that

$$\mathrm{Mr} = \frac{s(1+\varkappa a)^2}{a^2\varkappa(1+a)(1-\eta a^2)}\left\{\frac{2}{3}\eta a^3(1+\eta a)\left[\mathrm{Ga}(\rho^{-1}-1)+Wk^2\right] + 2aB(1+\eta a^3) + 2D_s(1+4\eta a+6\eta a^2+4\eta a^3+\eta^2 a^4)\right\}; \tag{5.1}$$

$$\omega^2 = B\,\frac{a^2k^4}{1+4\eta a+6\eta a^2+4\eta a^3+\eta^2 a^4}\left\{\frac{\eta a^2}{6}\left[\mathrm{Ga}(\rho^{-1}-1)+Wk^2\right] - D_s 2(1+\eta a^3)\right\} - D_s^2. \tag{5.2}$$

For comparison, we present the expression which determines the threshold value of Mr for the monotonic mode:

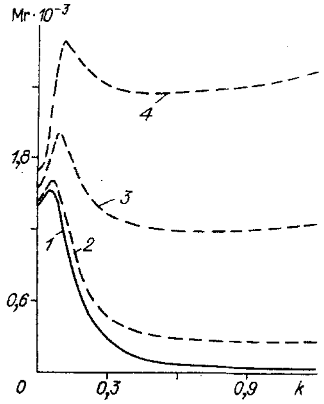

Fig. 5

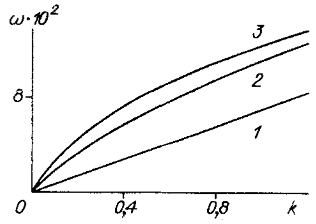

Fig. 6

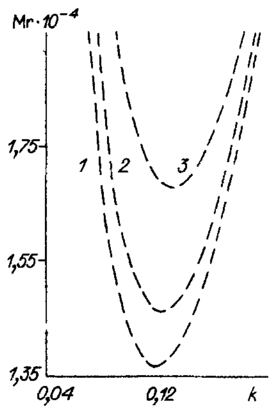

Fig. 7

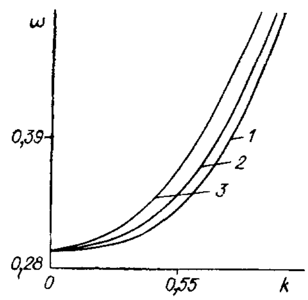

Fig. 8

$$\mathrm{Mr}_m = \frac{s\,(1+\varkappa a)^2}{\varkappa\,(1+a)\,(1-\eta a^2)}\left[\frac{2}{3}\,\eta a\,(1+\eta a)+\frac{B}{6D_s}\,\eta a^2\right][\mathrm{Ga}\,(\rho^{-1}-1)+Wk^2]. \tag{5.3}$$

Analysis of these expressions shows that in the range of a substantial oscillatory instability ($\omega^2 > 0$) $\mathrm{Mr} < \mathrm{Mr}_m$.

In order to "join" Eqs. (5.1)-(5.3) with formulas obtained [3] for long-wavelength perturbations with no interface deformation, we must investigate an intermediate region $k \sim W^{-1/4}$, in which we find

$$\mathrm{Mr} = s\,\frac{80\,(1+\eta a)\,(1+\varkappa a)^2}{\varkappa\,\mathrm{Pr}\,a^2\,(\chi a^2-1)}\,\frac{k^2}{k^4+k_*^4 S}; \tag{5.4}$$

$$\omega^2 = Ba^2k^2\left[L_1+(L_2+L_3k^4)\,(k^4+k_*^4 S)^{-1}\right]^{-1}, \tag{5.5}$$

where

$$k_*^4 = \frac{120\,(1+a)}{W\,\mathrm{Pr}\,\eta a^3}\left|\frac{1-\eta a^2}{\chi a^2-1}\right|;\quad S = \mathrm{sign}\left(\frac{1-\eta a^2}{\chi a^2-1}\right);$$

$$L_1 = \frac{a^2}{15}\left[(\eta+a\nu)+\eta\,(1+a)\,\frac{1-\nu a^2}{1-\eta a^2}\right];\quad L_2 = \frac{160\,(1+\eta)\,(1-\varkappa)\,(1-\eta a^2)}{\eta a\,(1+\varkappa a)\,W};$$

$$L_3 = -\frac{2\,a\,\mathrm{Pr}\,(1+\eta a^2)}{63\,(\chi a^2-1)\,(1+\varkappa a)}\Big\{\mathrm{Pr}\,[11\,(1-\varkappa\chi a^5)+53a\,(\varkappa-\chi a^3)+42\chi a^2\,(1-\varkappa a^2)]+$$
$$+\,(1+\varkappa a)\left[4\,(1-\chi\nu a^4)-\frac{21\,(1-\nu a^2)\,(1-\chi\eta a^4)}{10\,(1-\eta a^2)}\right]\Big\}.$$

If $S > 0$ for this system (as for systems studied in paragraph 4) then the oscillatory neutral curve is continuous and has a maximum at $k = k_*$. For $S < 0$, the oscillatory neutral curve has a discontinuity at $k = k_*$.

The long-wavelength asymptotic approximations of Eqs. (5.4) and (5.5) coincide with the short-wavelength asymptotic approximations of Eqs. (5.1) and (5.2). For $W^{-1/4} \ll k \ll 1$, Eqs. (5.4) and (5.5) transform to the corresponding formulas [3] derived when the interface deformation is neglected.

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